Chapter 4

Analysis of Variance

4.3 Tests for Two Means

Hypotheses About Mean Differences

Many research hypotheses compare two population means:

- Women live longer than men
- Republicans are more conservative than Democrats
- Happiness differs between old and young

Restate symbolically as population mean differences:

One-tailed:

$H_0: \mu_1 \leq \mu_2$

$H_1: \mu_1 > \mu_2$

Two-tailed:

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

Rearrange to show how the parameters differ from zero:

$H_0: \mu_1 - \mu_2 \leq 0$

$H_1: \mu_1 - \mu_2 > 0$

$H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 - \mu_2 \neq 0$

Apply the Central Limit Theorem

If large independent samples are drawn randomly from two populations, then the sampling distribution of their mean difference is also normally distributed

	POP #1	POP #2
Means	μ_1	μ_2
Std. Devs.	σ_1	σ_2
Sample Sizes	N_1	N_2

In the sampling distribution of mean differences:

Mean:

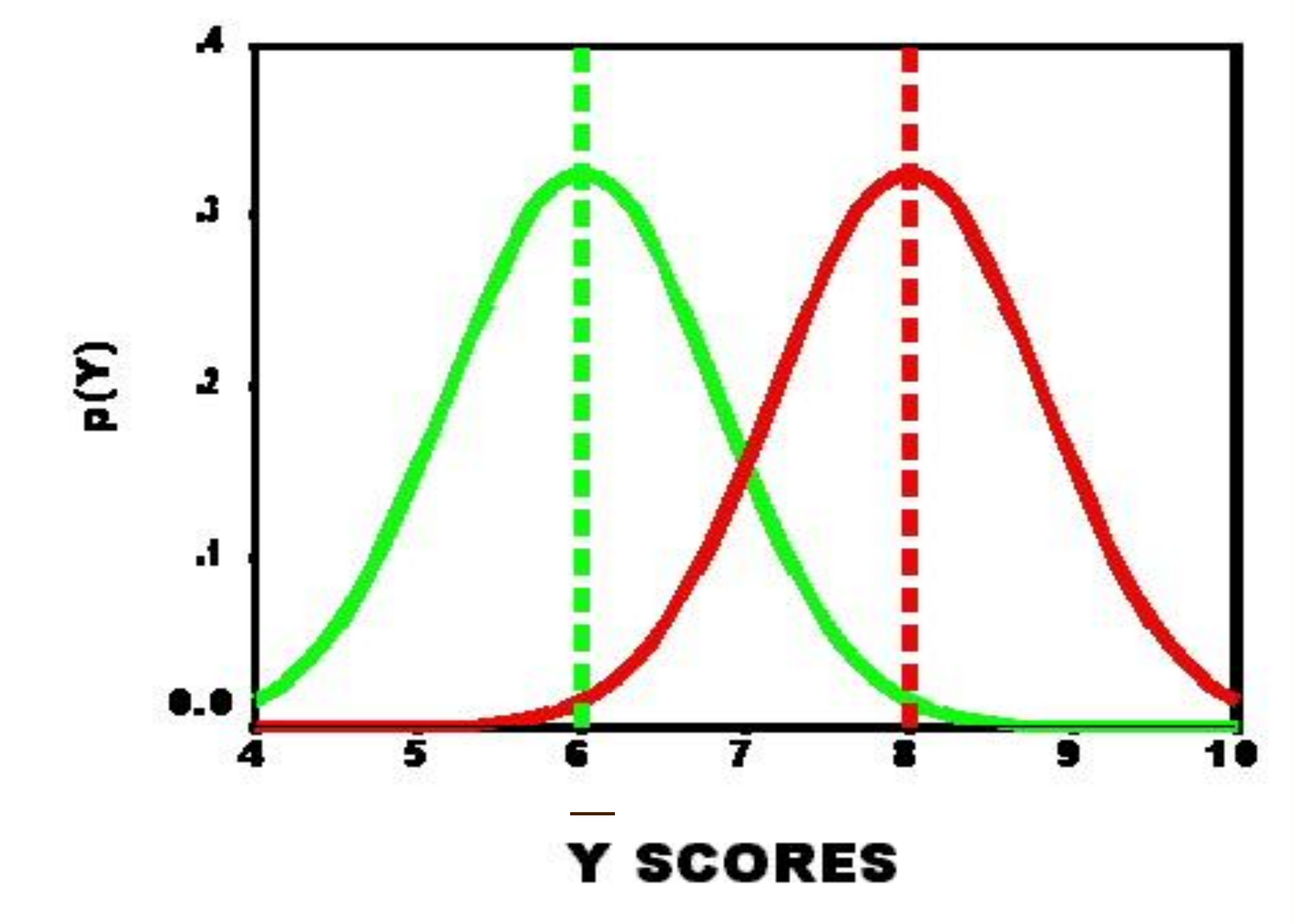
$$\mu_{(\overline{Y}_1 - \overline{Y}_2)} = \mu_1 - \mu_2$$

Standard error:

$$\sigma_{(\overline{Y}_1 - \overline{Y}_2)} = \sqrt{\sigma_1^2 / N_1 + \sigma_2^2 / N_2}$$

Dual sampling distributions

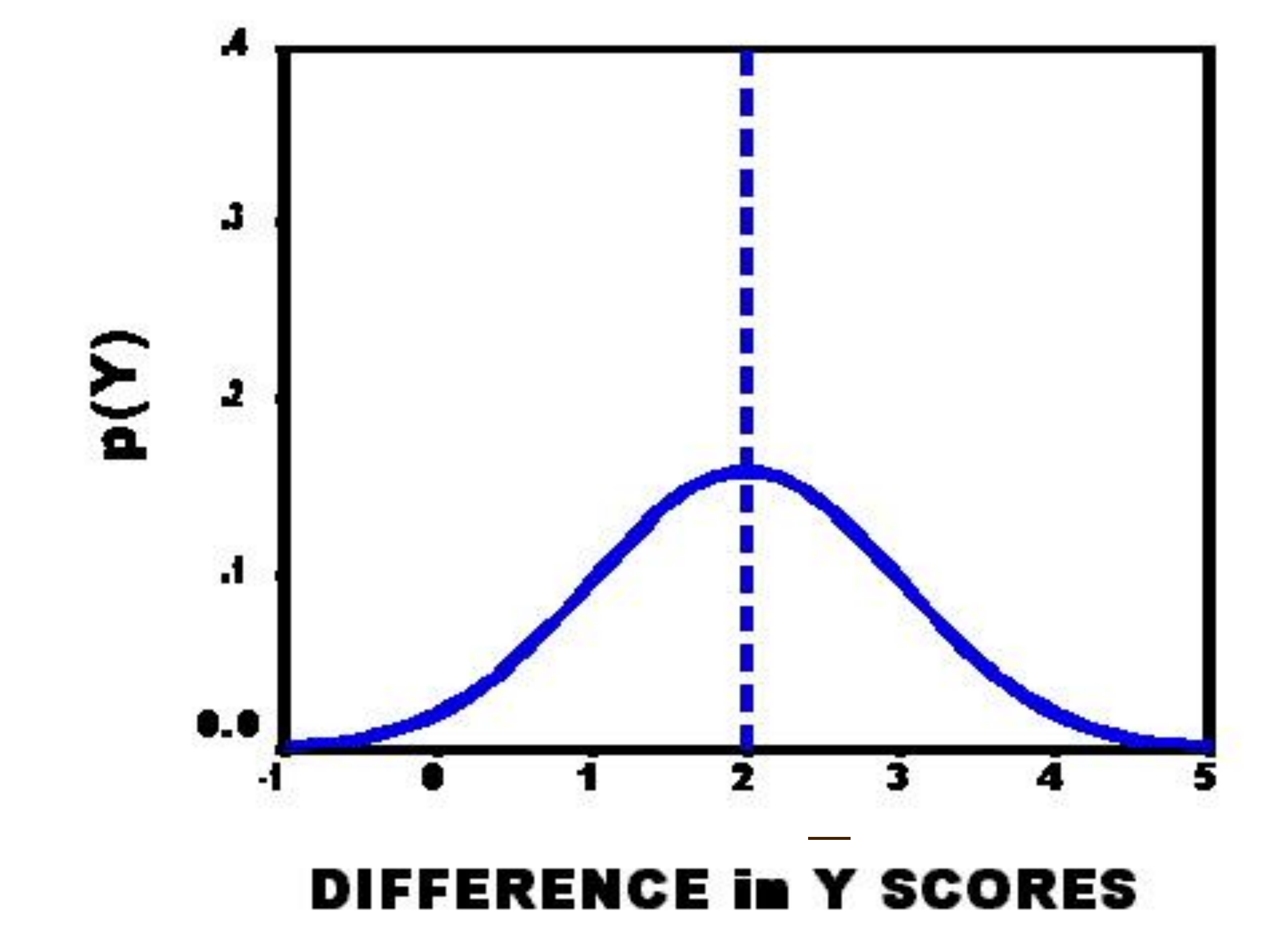
$\mu_1 - \mu_2 = 8 - 6 = +2$



Combined sampling distribution

Standard error of the mean difference is wider than the standard errors of the separate sampling distributions:

$$\sigma_{(\bar{Y}_1 - \bar{Y}_2)} = \sqrt{\sigma_1^2 / N_1 + \sigma_2^2 / N_2}$$



Null and Research Hypotheses

Translate these English statements into symbolic form null and research hypotheses. Determine whether a one-tailed or two-tailed test is required.

Older and younger people differ in church attendance

H_0: __________

H_1: __________

Women express more intense religiosity than men

H_0: __________

H_1: __________

Steps in Hypothesis Testing

1&2. State hypothesis pairs in English & symbolic forms; rearrange to show numerical difference in parameters

H_0: $\mu_1 = \mu_2$
H_1: $\mu_1 \neq \mu_2$

H_0: $\mu_1 - \mu_2 = 0$
H_1: $\mu_1 - \mu_2 \neq 0$

3. Choose α-level (Type I / false rejection error)

4. In the *Z* score table, find the critical value(s) necessary to reject H_0 at your chosen α-level

α (alpha)	One-tail c.v.	Two-tail c.v.
.05	1.65	±1.96
.01	2.33	±2.58
.001	3.10	±3.30

Steps (continued)

5. Estimate the standard error using <u>sample</u> statistics

$$\sigma_{(\bar{Y}_1 - \bar{Y}_2)} = \sqrt{\sigma_1^2 / N_1 + \sigma_2^2 / N_2} \quad \Rightarrow \quad \hat{\sigma}_{(\bar{Y}_1 - \bar{Y}_2)} = \sqrt{s_1^2 / N_1 + s_2^2 / N_2}$$

6. Calculate the *t*-test value, and compare it to the critical value(s); then decide whether to reject H_0

$$t = \frac{(\bar{Y}_1 - \bar{Y}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2 / N_1 + s_2^2 / N_2}}$$

Consistent with the null hypothesis, right-hand side of the numerator equals zero (see rearranged H_0)

7. If you reject H_0, what is probability of making a Type I error (false rejection error)? ________

8. State your substantive conclusion.

Test this research hypothesis with 2008 GSS data:

Older and younger people differ in church attendance

H_0: $\mu_O - \mu_Y = 0$

H_1: $\mu_O - \mu_Y \neq 0$

$$t = \frac{(\bar{\mathbf{Y}}_1 - \bar{\mathbf{Y}}_2) - (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)}{\sqrt{\mathbf{s}_1^2 / \mathbf{N}_1 + \mathbf{s}_2^2 / \mathbf{N}_2}}$$

	Old: 50-89 yrs	Young: 18-49 yrs
N	897	1,108
Mean	26.3	18.3
Variance	753.1	588.9

= ____

Decision about null hypothesis: ____________________

Probability of Type I error: ________________________

Conclusion: ____

Test this research hypothesis with 2008 GSS data:

Women & men differ in newspaper reading (times per year)

H_0: $\mu_W = \mu_M$

H_1: $\mu_W \neq \mu_M$

$$t = \frac{(\overline{Y}_1 - \overline{Y}_2) - (\mu_1 - \mu_2)}{\sqrt{s_1^2 / N_1 + s_2^2 / N_2}}$$

	Women	Men
N	701	628
Mean	175.3	179.6
Variance	21,786.0	22,398.0

= ______________________________

Decision about null hypothesis: ______________________

Probability of Type I error: ______________________

Conclusion: ______________________

Test this research hypothesis with 2008 GSS data:

In the past 5 years, men had more sex partners than women

H_0: $\mu_M \leq \mu_W$

H_1: $\mu_M > \mu_W$

	Women	Men
N	933	775
Mean	1.8	4.6
Variance	18.6	187.7

$$t = \frac{(\bar{\mathbf{Y}}_1 - \bar{\mathbf{Y}}_2) - (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)}{\sqrt{\mathbf{s}_1^2 / \mathbf{N}_1 + \mathbf{s}_2^2 / \mathbf{N}_2}}$$

= ______________________________

Decision about null hypothesis: ______________________

Probability of Type I error: __________________________

Conclusion: ______________________________________

Hypotheses About Proportions

For dichotomous dependent variables, form hypotheses about population differences in two proportions

- Liberals favor legalizing pot more than conservatives

One-tailed:

H_0: $\rho_L \leq \rho_C$
H_1: $\rho_L > \rho_C$

Rearranged:

H_0: $\rho_L - \rho_C \leq 0$
H_1: $\rho_L - \rho_C > 0$

- Blacks and whites differ in support for death penalty

Two-tailed:

H_0: $\rho_B = \rho_W$
H_1: $\rho_B \neq \rho_W$

Rearranged:

H_0: $\rho_B - \rho_W = 0$
H_1: $\rho_B - \rho_W \neq 0$

Standard Error of Dichotomous Proportions

A proportion is the relative frequency of one outcome to the total sample size. The two proportions of a dichotomy sum to unity:

$$p_0 = \frac{f_0}{N} \qquad p_1 = \frac{f_1}{N} \qquad \text{and } p_0 + p_1 = 1.00$$

(In B&K, p. 124: p_1 = p and p_0 = q, thus p + q = 1.00)

Estimated standard error for <u>one</u> population's sampling distribution:

$$s_p = \sqrt{p_0 p_1 / N} = \sqrt{pq / N}$$

Estimated standard error for the <u>difference</u> in <u>two</u> populations' sampling distributions:

$$s_{p_1 - p_2} = \sqrt{\frac{p_1 q_1}{N_1} + \frac{p_2 q_2}{N_2}}$$

Test this research hypothesis with 2008 GSS data:

Liberals visit art museums more than conservatives

H_0: $\rho_L - \rho_C \leq 0$

H_1: $\rho_L - \rho_C > 0$

$$t = \frac{(\mathbf{p_L} - \mathbf{p_C}) - (\rho_L - \rho_C)}{\sqrt{\mathbf{p_L q_L} / \mathbf{N_L} + \mathbf{p_C q_C} / \mathbf{N_C}}}$$

	Libs	Cons
N	397	501
Prop. p	.86	.60
Prop. q	.14	.40

= ______________________________

Decision about null hypothesis: ______________________________

Probability of Type I error: ______________________________

Conclusion: ______________________________

Test this research hypothesis with 2008 GSS data:

Protestants and Catholics differ on "abortion for any reason"

H_0: $\rho_P - \rho_C = 0$

H_1: $\rho_P - \rho_C \neq 0$

$$t = \frac{(p_P - p_C) - (\rho_P - \rho_C)}{\sqrt{p_P q_P / N_P + p_C q_C / N_C}}$$

	Protestant	Catholic
N	676	295
Prop. p	.35	.40
Prop. q	.65	.60

= ______________________________

Decision about null hypothesis: ____________________

Probability of Type I error: ________________________

Conclusion: ________________________________

Hypotheses About Paired Means

Sometimes researchers want to compare the means of:

(1) two matched samples, such as husbands and wives

(2) the same person's responses to one variable at two times (e.g., before and after some experience)

(3) two variables measured on identical scales for each person

- **Who does more housework, husbands or wives?**
- **Do you feel about statistics today as you did last month?**
- **Which tastes better – Coke or Pepsi?**
- **Do Americans like Japan or China more?**

We can't apply the two-sample t-test. Although the sample size is 2N cases (paired scores from N cases), the members of each pair were not selected independently.

Instead, calculate *t* with the difference in paired scores.

Hypotheses about paired means ask whether the difference is zero in the two populations: $\mu_D = \mu_1 - \mu_2$

H_0: $\mu_D = 0$

H_1: $\mu_D \neq 0$

In a sample, the difference for a pair of scores is:

$$Y_D = Y_{1i} - Y_{2i}$$

Calculate the sample standard deviation of the differences:

$$s_D = \sqrt{\frac{(Y_D - \bar{Y}_D)^2}{N-1}}$$

Then estimate the standard error:

$$s_D / \sqrt{N}$$

Test this research hypothesis with 2008 GSS data:

Americans differ in their level of confidence in business and confidence in Congress, each measured on a 5-point scale.

H_0: $\mu_D = 0$

H_1: $\mu_D \neq 0$

	Business	Congress
Mean	2.91	2.57
Sample N	1,333	
s_D	0.94	

$$t = \frac{\bar{Y}_D - \mu_D}{s_D / \sqrt{N}}$$

= ____

Decision about null hypothesis: ______________________

Probability of Type I error: ___________________________

Conclusion: ____________________________________