Chapter 3

Making Statistical Inferences

3.7 The *t* Distribution

3.8 Hypothesis Testing

3.9 Testing Hypotheses About Single Means

Limitations of the Normal Distribution

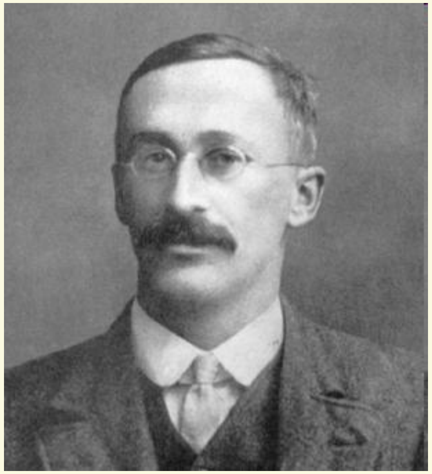
Researchers want to apply the central limit theorem to make inferences about population parameters, using one sample's descriptive statistics, by applying the theoretical standard normal distribution (*Z* table).

But, except in classroom exercises, researchers will <u>never</u> actually know a population's variance. Hence, they can calculate neither the sampling distribution's standard error nor the *Z* score for any sample mean!

So, have we hit a statistical dead-end? ______

The *t* distribution

Fortunately, we can use "Student's *t* distribution" (created by W.S. Gossett, a quality control expert at the Guinness Brewery) to estimate the unknown population standard error from the sample standard deviation.



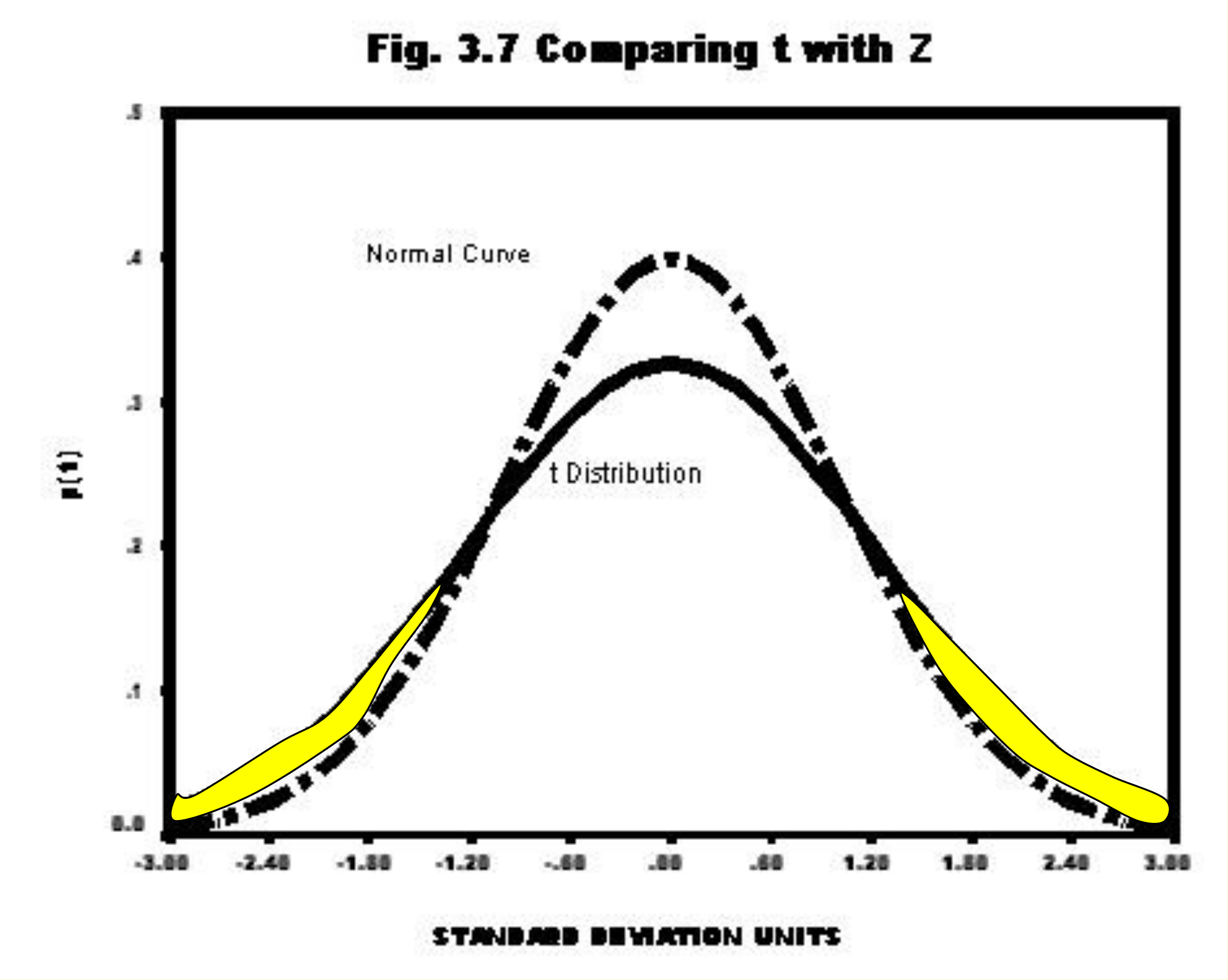
Z-scores and t-scores for the ith sample are very similar:

$$Z_i = \frac{\bar{Y}_i - \mu_Y}{\sigma_{\bar{Y}}} \qquad t_i = \frac{\bar{Y}_i - \mu_Y}{s_Y / \sqrt{N}}$$

- *Z* score uses the standard error of a population
- *t* score uses a sample estimate of that standard error

Z versus *t* distributions

Generally, t distributions have thicker “tails”



A Family of *t* distributions

The thickness of the tails depends on the sample size (*N*).

Instead of just one *t* distribution, an entire family of *t* scores exists, with a different curve for every sample size from N = 1 to ∞ .

Appendix D gives *t* family's critical values

But, for large samples (*N* > 100), the *Z* and *t* tables have nearly identical critical values.

(Both tables' values are identical for *N* = ∞)

Therefore, to make statistical inferences using large samples – such as the 2008 GSS (*N* = 2,023 cases) – we can apply the standard normal table to find *t* values!

Hypothesis Testing

The basic statistical inference question is:

What is the probability of obtaining a sample statistic if its population has a hypothesized parameter value?

H_1: Research Hypothesis - states what you really believe to be true about the population

H_0: Null Hypothesis - states the opposite of H_1; this statement is what you expect to reject as untrue

Scientific positivism is based on the logic of falsification -- we best advance knowledge by disproving null hypotheses.

We can never prove our research hypotheses beyond all doubts, so we may only conditionally accept them.

Writing Null & Research Hypotheses

Hypotheses are always about a population parameter, although we test their truth-value with a sample statistic. We can write paired null and research hypotheses in words and in symbols.

A: Hypotheses stating the direction of a population mean

H_0: U of MN student GPA is below 3.00

H_1: U of MN student GPA is 3.00 or higher

$$H_0: \mu < 3.00 \text{ GPA}$$
$$H_1: \mu \geq 3.00 \text{ GPA}$$

B: Hypotheses uncertain about population mean's direction

H_0: U of MN student GPA is 3.00

H_1: U of MN studnt GPA is not 3.00

$$H_0: \mu = 3.00$$
$$H_1: \mu \neq 3.00$$

Errors in Making Inferences

We use probability theory to make inferences about a population parameter based on a statistic from a sample.

But, we always run some risk of making an incorrect decision -- we might draw an extremely unlikely sample from the tail of the sampling distribution.

Type I error (false rejection error) occurs whenever we incorrectly reject a true null hypothesis about a population

Suppose that a sample of UMN student GPAs is 3.16. Based on the only evidence available to us, we decide to REJECT the null hypothesis H_0 in the first pair above.

But, in the population (unknown to us), the true UMN student GPA is 2.97. Thus, our decision to reject a null hypothesis that really is true would be an ERROR. We should try to make the chances of making such false rejection errors as small as possible.

Type I & Type II Errors

BOX 3.2.		(2) Based on the <u>sample</u> results, you must decide to:	
		Reject null hypothesis	**Do not reject H_0**
(1) In the <u>population</u> from which that sample came, the null hypothesis H_0 really is:	**True**	Type I or false rejection error (α)	Correct decision
	False	Correct decision	Type II or false acceptance error (β)

Choosing the probability of Type I error

The probability of making a decision that results in a false rejection error (Type I error) is **alpha (α).**

We can choose an alpha area, in one or both tails of a sampling distribution, called the **Region of Rejection**.

Three conventional alpha areas that statisticians choose for probability of a Type I error are:

α **= .05**

α **= .01**

α **= .001**

As a researcher, you control the size of Type I error by choosing how big or small a risk you're willing to take if you make a wrong decision. How much is at stake if you're wrong?

When you choose an α, you must live with consequences of your decision.

How big a risk would you take of falsely rejecting H_0 that an AIDS vaccine is "unsafe to use"?

One- or Two-Tailed Tests?

How can you decide whether to write
a one-tailed research hypothesis --

or a two-tailed
research hypothesis?

You can use social theory, past research results, or even your hunches to choose your hypotheses that reflects the most likely current state of knowledge:

Two tail: states a difference, but doesn't say where

One tail: states a clear directional difference

Turning Off Highway Ramp Meters

In 2001, the Minnesota Legislature ordered all 430 ramp meter lights turned off for 6 weeks, a natural experiment about effects of metering on travel time, crashes, driver satisfaction. In the same period one year before, a total of 261 vehicle crashes occurred.

What are possible 1- and 2-tailed hypotheses that could be tested?

Politician: Turning off ramp meters will reduce traffic crashes

$$H_0: \mu_Y \geq 261 \text{ crashes}$$
$$H_1: \mu_Y < 261 \text{ crashes}$$

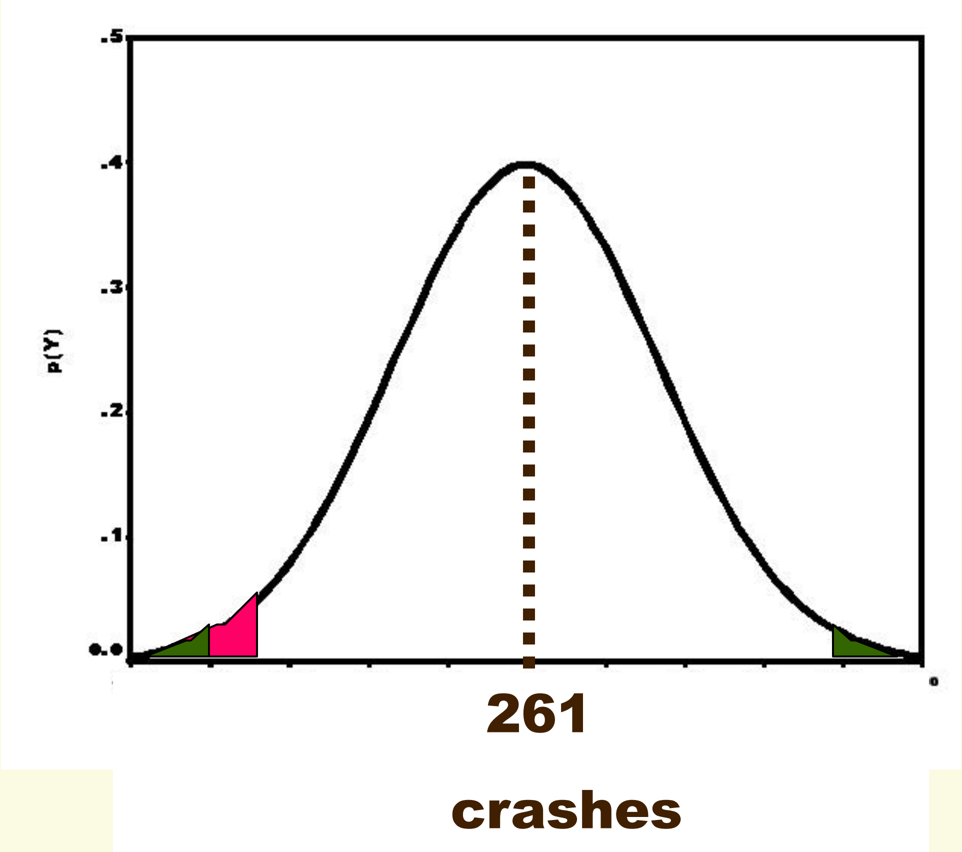
Engineer: Turning off meters will change the number of traffic crashes, but they might either increase or decrease

$$H_0: \mu_Y = 261 \text{ crashes}$$
$$H_1: \mu_Y \neq 261 \text{ crashes}$$

Which alpha regions for which hypotheses?

Politician's prediction is probably true if the sample mean falls into which region(s) of rejection?

Engineer's prediction is probably true if the sample mean falls into which region(s) of rejection?



An evaluation found that crashes increased to 377 with the meters turned off, a jump of 44%! Also, traffic speed decreased by 22% and travel time became twice as unpredictable due to unexpected delays. Any question why ramp meters were turned back on after six weeks?

Box 3.4 Significance Testing Steps

1. State a research hypothesis, H_1, which you believe to be true.

2. State the null hypothesis, H_0, which you hope to reject as false.

3. Chose an α-level for H_0 It designates the region(s) of rejection, which is the probability of Type I error (a.k.a. false rejection error)

4. In the normal *(Z)* table, find the critical value(s) (c.v.) of *t*

5. Calculate *t* test statistic from the sample values:

   - Use the sample s.d. and *N* to estimate the standard error

6. Compare this *t*-test statistic to the c.v. to see if it falls inside or outside the region(s) of rejection (draw a sampling distribution)

7. Decide whether to reject H_0 in favor of H_1; if you reject the null hypothesis, state the probability α of making a Type I error

8. State a substantive conclusion about the variables involved

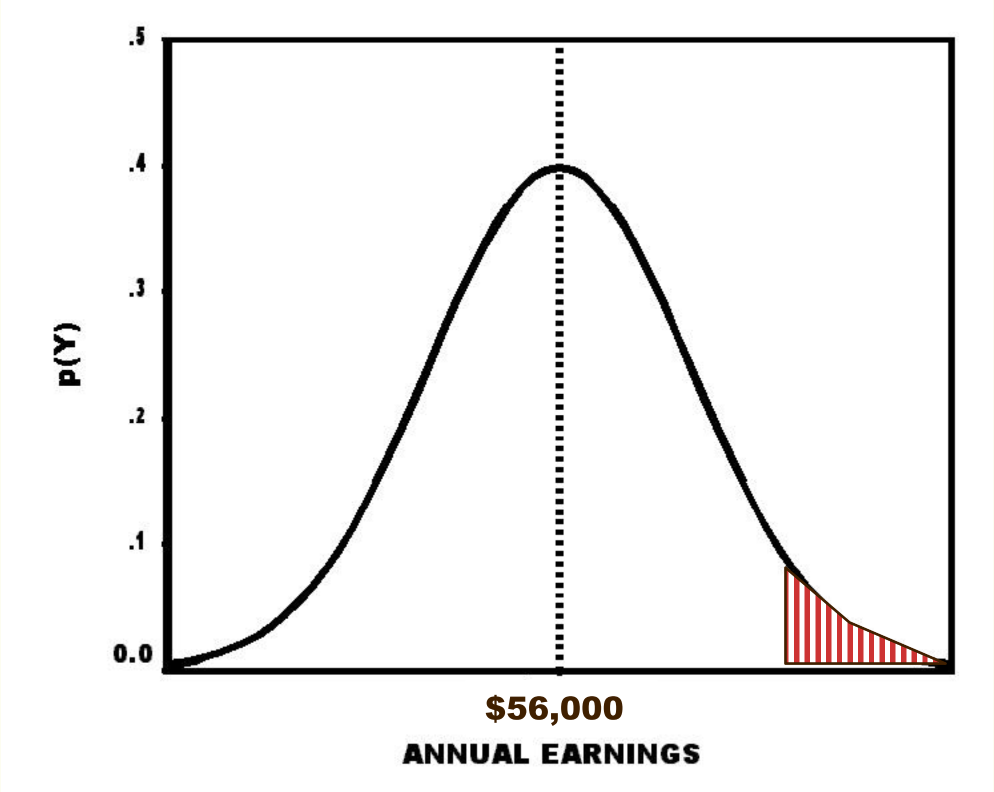
Let’s test this pair of hypotheses about American family annual earnings with data from the 2008 GSS:

H_0: American family incomes were $56,000 or less

H_1: American family incomes were <u>more than</u> $56,000

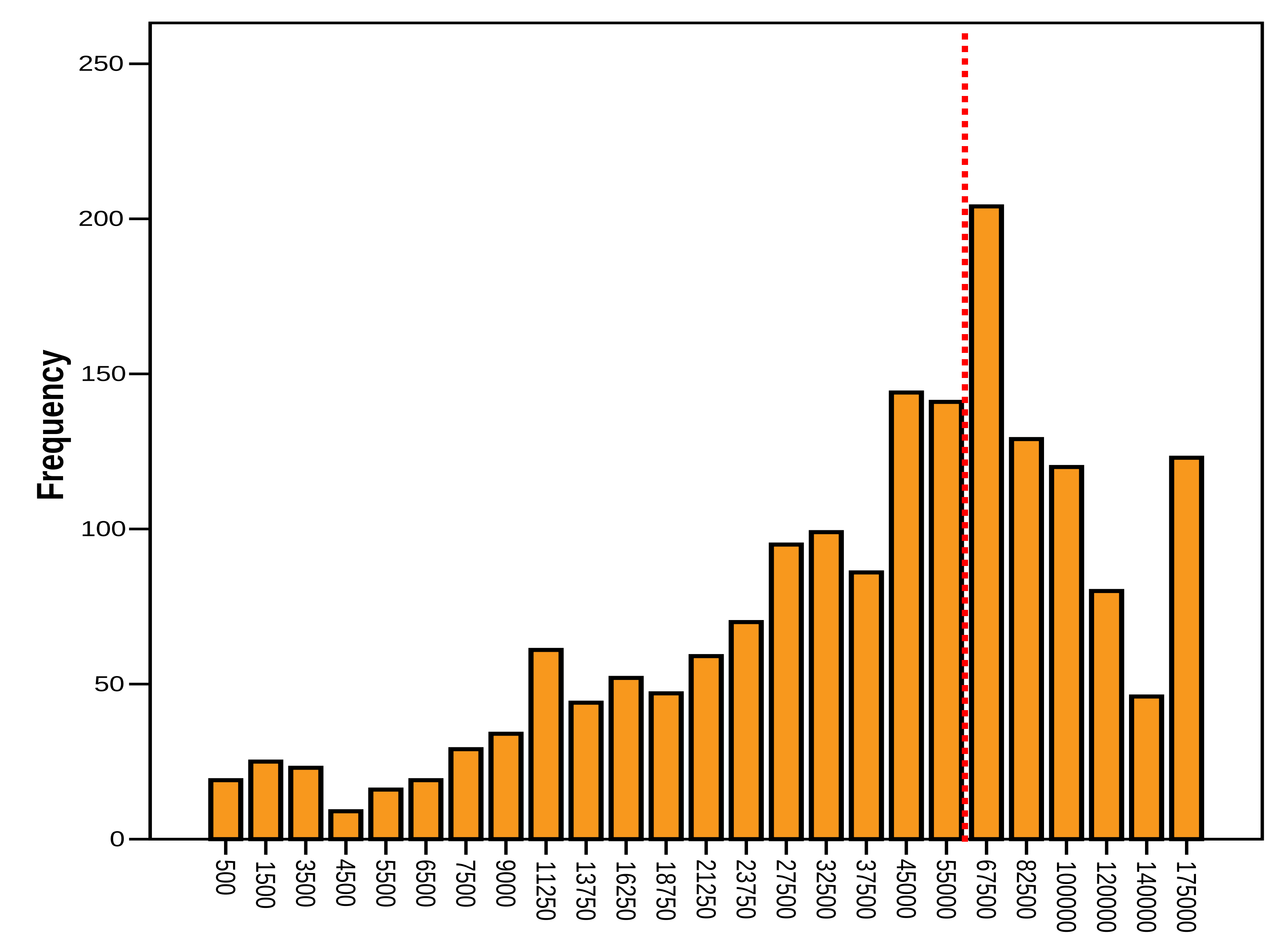
H_0: $\mu_Y \leq$ \$56,000
H_1: $\mu_Y >$ \$56,000

H_1 puts the region of rejection (α) into the <u>right-tail</u> of the sampling distribution which has mean earnings of $56,000:



2008 GSS Sample Statistics on Income

Mean = $58,683   Stand. dev. = $46,616   N = 1,774



Perform the *t*-test (*Z*-test)

3. Choose a medium probability of Type I error: **α = .01**

4. What is c.v. of t_α? (in C Area beyond Z) __________

5. Compute a *t* test statistic, using the sample values:

$$t = \frac{\overline{\mathbf{Y}} - \mu_{\mathbf{Y}}}{s_{\mathbf{Y}} / \sqrt{\mathbf{N}}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

6-7. Compare *t*-test to c.v., then make a decision about H_0 :

If this test statistic fell into the region of rejection, you must decide to ____________ the null hypothesis.

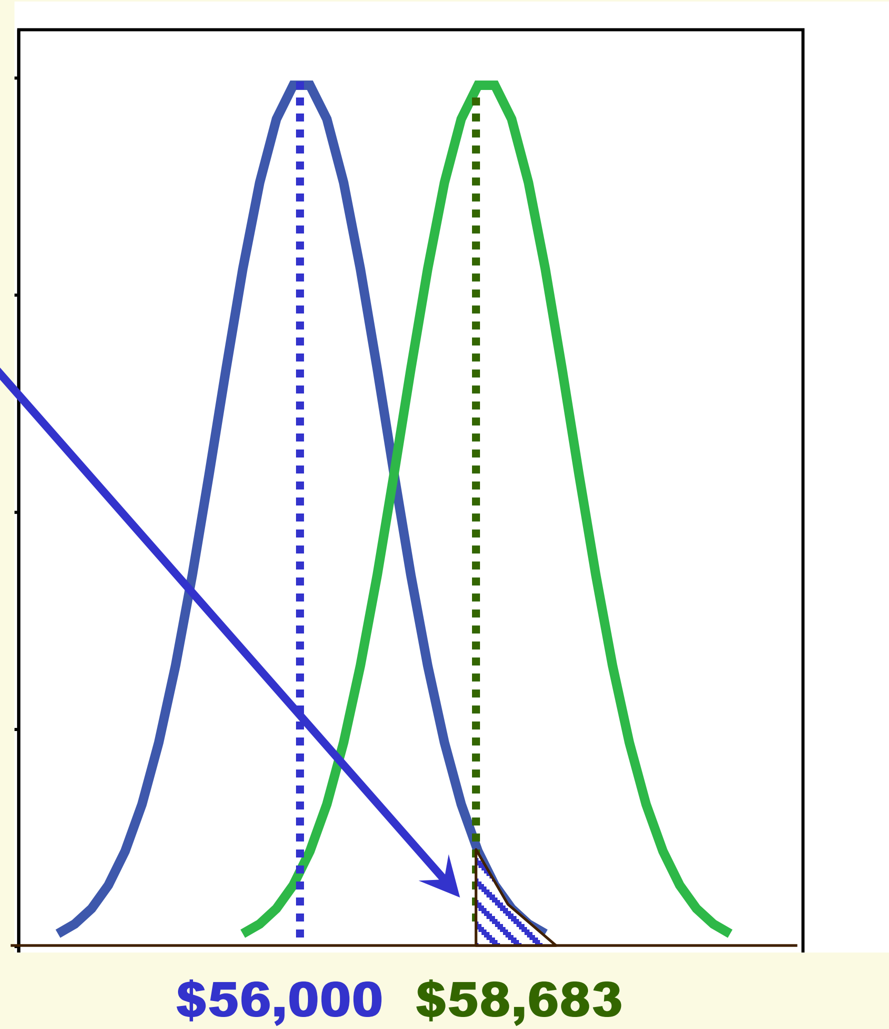
The Probability of Type I error is ___________.

8. Give a substantive conclusion about annual incomes:

____

In the left (blue) sampling distribution, whose mean income = $56,000, the region of rejection overlaps with another sampling distribution (green), which has higher mean income = $58,683.

Thus, although the 2008 GSS sample had a low probability ($p < .01$, the shaded blue alpha area) of being drawn from a population where the mean income is $56,000, that sample had a very high probability of coming from a population where the mean family income = $58,683.



A researcher hypothesizes that, on average, people have sex <u>more than</u> once per week (52 times per year)

Write a one-tailed hypothesis pair: **H_0: $\mu_Y \leq 52$**
H_1: $\mu_Y > 52$

Set α = .001 and find critical value of *t*: ______________

Sample statistics: Mean = 57.3; st. dev. = 67.9; *N* = 1,686

Estimate standard error and the *t*-test:

$$t = \frac{\bar{Y} - \mu_Y}{s_Y / \sqrt{N}}$$ ______________________________________

Compare *t*-score to c.v., decide H_0: __________________

What is probability of Type I error? __________________

Conclusion: ______________________________________

A researcher hypothesizes that the proportion of people getting news from the Internet differs from 0.30.

Write a two-tailed hypothesis pair: **H_0: ρ = 0.30**
H_1: ρ ≠ 0.30

Set $\alpha = .01$ and find c.v. for *t*-test: ________________

Proportion = 0.28; standard error = 0.012 N = 1,376

Estimate standard error and the *t*-test:

$$t = \frac{p - \rho}{s_{\bar{p}}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Compare *t*-score to c.v., decide H_0: ________________

What is probability of Type I error? ________________

Conclusion: ______________________________________

A research hypothesis is that people visit bars more than once per month (12 times/year)

Write a one-tailed hypothesis pair: **H_0: $\mu_Y \leq 12$**
H_1: $\mu_Y > 12$

Set α = .01 and find c.v. for *t*-test: ________________

Mean = 16.4; st. dev. = 42.9; *N* = 1,328

Estimate standard error and the *t*-test:

$$t = \frac{\bar{Y} - \mu_Y}{s_Y / \sqrt{N}} = \rule{10cm}{0.4pt}$$

Compare *t*-score to c.v., decide H_0: ________________

What is probability of Type I error? ________________

Conclusion: ______________________________________

A demographer hypothesizes that the mean number of people living in U.S. households is now below 2.50?

Write a one-tailed hypothesis pair: **H_0: $\mu_Y \geq 2.50$**

H_1: $\mu_Y < 2.50$

Set α = .001 and find c.v. for *t*-test: ________________

Mean = 2.47 people; st.dev. = 1.42; *N* = 2,023

Estimate standard error and the *t*-test:

$$t = \frac{\bar{\mathbf{Y}} - \mu_{\mathbf{Y}}}{s_{\mathbf{Y}} / \sqrt{\mathbf{N}}} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Compare *t*-score to c.v., decide H_0: ________________

What is probability of Type I error? ________________

Conclusion: ______________________________________

Can we reject the null hypothesis that the mean church attendance is twice per month (24 times/year)?

Write a two-tailed hypothesis pair: $H_0: \mu_Y = 24$

$H_1: \mu_Y \neq 24$

Set $\alpha = .001$ and find c.v. for *t*-test: ____________________

Mean = 21.9 times/year; st.dev. = 26.0; *N* = 2,014

Estimate standard error and the *t*-test:

$$t = \frac{\bar{Y} - \mu_Y}{s_Y / \sqrt{N}} = \underline{\hspace{10cm}}$$

Compare *t*-score to c.v., decide H_0: ____________________

What is probability of Type I error? ____________________

Conclusion: ____